

# Properties of 2020

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## Abstract

This document provides some properties of 2020. If you have a good one, feel free to let me know! Also, note that the properties in each section go from easy to interesting to overkill. Note that the ones that look weird and/or are overkill are likely from OEIS, so I'm not weird OEIS is.

## 1 Algebra

### 1.1 Operations on 2020

$2020^2$  is 4080400 and  $2020^3$  is 8242408000.  $\sqrt{2020}$  is 44.9444101085 and  $\sqrt[3]{2020} = 12.6410686485$ .  $\ln 2020$  is 7.6108527903953 and  $\log_{10} 2020$  is 3.3053513694466.  $\sin 2020$  is 0.044061988343923,  $\cos 2020$  is -0.99902879897588, and  $\tan 2020$  is -0.044104822993183.  $\frac{1}{2020}$  has period 4.

### 1.2 Time

2020 seconds is equal to 33 minutes, 40 seconds. 2020 is a leap year. 2020 has 53 Wednesdays and Thursdays. The previous year this occurs was 1992, and the next is 2048.

### 1.3 Triangular Numbers

2020 is equal to  $\sum_{k=3}^{22} T_k$ , where  $T_k$  is the  $k$ th triangular number. Also,  $T_5 + T_{30} + T_{55} = 2020$ .

### 1.4 Heptagonal Numbers

The alternating sum of the first 40 heptagonal numbers is 2020.

### 1.5 Sequences

A sequence that starts with 4 then adds 1, then 2, then 3, and so on contains 2020. Also, in the product  $f(x) = \prod_{k=1}^{\infty} (1 + 4 \cdot x^k)$ ,  $x^{14}$  has coefficient 2020. Also, in the expansion of  $\frac{\sum_{i=1}^{\infty} \frac{x^{i^2}}{1-x^{i^2}}}{\prod_{i=1}^{\infty} \frac{x^{i^2}}{1-x^{i^2}}}$ , the coefficient of  $x^{51}$  is 2020. Also, if  $a_0 = 4$ , and for  $n > 0$ ,  $a_n = a_{n-1} + 2^n - 3$ , then  $a_{10} = 2020$ . In the expansion of  $\prod_{k=1}^7 \frac{1}{1-x^{2^k-1}}$ , the coefficient of  $x^{53}$  is 2020. Also, in the expansion of  $\frac{1}{1-x^3-x^4-x^5-x^6}$ , the coefficient of  $x^{28}$  is 2020. Also, in the expansion of  $\prod_{k=1}^{\infty} \frac{1}{1+x^k}^{k-1}$ , the coefficient of  $x^{35}$  is -2020.

### 1.6 Trigonometry

The generating function of  $\frac{\tan x \cdot \sin(\tanh x)}{2}$  has -2020 as the coefficient of  $x^8$ .

## 1.7 Doublets

2020 is the juxtaposition of two identical strings of 20.

## 1.8 Sperner Systems

The number of monotone Boolean functions of 5 variables with 4 mincuts is 2020.

## 1.9 Ramanujan's Theta Function

The 52nd term of the expansion of  $\frac{1}{f(-x, -x^5)}$  is 2020.

## 1.10 Hermite Polynomials

The numerator of  $H(3, \frac{1}{13})$  is -2020, where  $H$  is a Hermite polynomial.

## 1.11 Pythagorean Approximations

The denominators of the Pythagorean Approximations of 2 to 5 is 2020.

# 2 Geometry

## 2.1 Outer Vecten Triangle

The outer vecten triangle of an integer triangle has area 2020. (Note: I need help finding which integer triangle this is!)

## 2.2 Convolution Triangle of A Fibonacci-Like Sequence

The convolution triangle of  $a_n$ , where  $a_n = 2 \cdot (a_{n-1} + a_{n-2})$  has element 2020 in the 40th position.

# 3 Combinatorics

## 3.1 Catalan Numbers

$32_{10}$  is  $2020_C$ , where base  $C$  is base Catalan Numbers.

## 3.2 Bernoulli Numbers

$B_{2020}$  has denominator 330, along with 20, 340, 1220, 1420, 2020, 2980, 3340, 3940, 4460, 4540, 4580, 5140, 5660, 5780.

## 3.3 Partitions

The number of strict integer partitions of 58 not containing 1 or any part whose prime indices all belong to the partition is 2020. Also, the number of integer partitions of 44 whose augmented differences are weakly decreasing is 2020. Also, the number of overcubic partions of 12 is 2020. Also, the number of partions of 32 such that the sum of the distinct odd parts is greater than  $\frac{n}{2}$  is 2020. Also, the number of partitions of 53 into Heegner numbers is 2020. Also, the number of ordered partitions (i.e. compositions) of 24 into 4 relatively prime parts is 2020. Also, the number of compositions of 21 such that no two adjacent parts are equal is 2020.

### 3.4 Grids

The number of lines going through exactly 8 points in a 4040 grid of points is 2020.

### 3.5 Tessellations

The number of 1-sided polycairos with 9 cells is 2020.

### 3.6 Symmetric Group

The number of character table entries of the symmetric group  $S_{12}$  greater than 0 is 2020. (Note that the idea of  $S_n$  is not completely useless to competition - it describes a set of numbers mapping to themselves, which is an idea featured in the last few combinatorics questions of HMMT!)

### 3.7 Numbers in a Range

The number of even numbers in the range  $10n$  to  $10n + 9$  where  $n = 40$  is 2020.

### 3.8 Latin Rectangles

The number of 36 Latin rectangles in which the second row contains 2 cycles with the same order of elements is 2020.

### 3.9 Graphs

The number of nodes in the 12th level of the Euclid-Mullin Graph starting with 1 is 2020.

### 3.10 Conway's Game of Life

The number of active (ON, black) cells in 39th stage of growth of two-dimensional cellular automaton defined by "Rule 118", based on the 5-celled von Neumann neighborhood is 2020.

### 3.11 Fredholm-Rueppel Inverse Triangle

In the Fredholm-Rueppel Inverse Triangle, the 42nd and 43rd row sums are both -2020.

## 4 Number Theory

### 4.1 Prime Factorization

The prime factorization of 2020 is

$$2^2 \cdot 5 \cdot 101.$$

From here we can deduce 2020 is composite and even.

### 4.2 Number of Primes

The exist 3 distinct prime factors: 2, 5, 101.

### 4.3 Primes

Prime numbers close to 2020 include: 2011, 2017, 2027, 2029. The 2020th prime is 17573. Also, the sum of the primitive roots of the 52nd prime (239) is 2020. Also, the sequence in which any two consecutive digits in the sequence sum up to a prime (starting with 1) contains 2020. Also, the 15th prime multiplied by the 14th prime is 1 more than 2020 (15th prime is 47 and 14th prime is 43). Also,  $6 \cdot 2020^2 - 1$  and  $6 \cdot 2020^2 + 1$  are twin primes. Also, in the set of the number of positive integers  $z$  such that  $\pi(x^3 + y^3) = \pi(z^3)$  for some  $0 < x \leq y \leq z$ , 2020 is included. Also, the number of primes under  $26^3$  is 2020. Also,  $k = 2020$  is the smallest integer  $k$  such that  $\frac{n! - k}{n}$  is prime, where  $n = 20$ . Also, for the set of numbers  $n$  such that  $n^{1024} + (n + 1)^{1024}$ , 2020 is included in that set. 202099 is prime. Also, the average of the 2020th prime number and the 2021st prime number is a perfect cube. Also, the 2020th prime minus 2020 is a brilliant number (15553).

### 4.4 Number of Divisors

There are 12 divisors: 1, 2, 4, 5, 10, 20, 101, 202, 404, 505, 1010, 2020.

### 4.5 Sum of Divisors

The sum of the divisors of  $n$  is 4284. The sum of the proper divisors (i.e. not including 2020) is 2264.

### 4.6 Product of Divisors

The product of the divisors of 2020 is  $2020^6$ .

### 4.7 Abundance

2020 is an abundant number with abundance 244. Also, 2020, 2024, 2026 are an abundant triple (all are abundant).

### 4.8 Bases

2020 in binary is  $11111100100_2$ . 2020 in hexadecimal is  $7E4_{16}$ . 2020 is a palindrome in base 21:  $4C4_{21}$ . It is a pernicious number because it has 7 ones in binary (pernicious means it has an odd number of ones in binary).  $2020_4$  is a self-descriptive number, where a self-descriptive number is an integer  $m$  that in a given base  $b$  is  $b$  digits long in which each digit  $d$  at position  $n$  (the most significant digit being at position 0 and the least significant at position  $b - 1$ ) counts how many instances of digit  $n$  are in  $m$ .

### 4.9 Digits

2020 has 4 digits. The sum of digits of 2020 is 4, which makes it a Harshad Number (divisible by the sum of its digits).

### 4.10 Squares and Sum of Squares

2020 has 2 representations as a sum of 2 squares:  $2020 = 16^2 + 42^2 = 24^2 + 38^2$ . Also, note that  $2020 = 17^2 + 19^2 + 23^2 + 29^2$ , where these are the 7th, 8th, 9th, and 10th prime numbers respectively. The smallest square less than 2020 is 1936. It can be written as the sum of 10 consecutive even squares:  $4^2 + 6^2 + \dots + 20^2 + 22^2 = 2020$ . Also, (45, 2020) is a lattice point of  $y = x^2 - 5$ .

#### 4.11 Factorial

$2020! = 2^{2013} 3^{1005} 5^{503} 7^{334} \dots$

#### 4.12 Exponentiation

$2020^3 | k^{2020^2} - 1$  for  $k = 3, 9, 11$ . 2020 is equal to  $2^{11} - 28$ , and 2020 divides  $91^4 - 1$  (note that this also bleeds into divisibility). Also,  $2020^3 | 3^{2020^2} - 1$ . Also,  $2020^3 | 11^{2020^2} - 1$ . Also,  $\lfloor 21^{\frac{5}{2}} \rfloor = 2020$ .  $2020, 2020^2, 2020^3$  all only use even digits.

#### 4.13 Modulos

$2020 \pmod{m}$  is  $0, 1, 0, 0, 4, 4, 4, 4$  for  $m = 1, 2, 3, 4, 5, 6, 7, 8, 9$  respectively. Also,  $1616^k \equiv 1616 \pmod{2020}$  for all  $k \geq 1$ .

#### 4.14 Fibonacci

The 8-step Fibonacci sequence,  $a_n = \sum_{k=1}^7 a_{n-k}$ , has 19th term 2020. Also, in the 3-step Fibonacci sequence (a.k.a Tribonacci),  $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ ,  $a_{2020}$  is a prime.

#### 4.15 Roman Numerals

2020 is MMXX in Roman Numerals.

#### 4.16 Totient Function

The number of positive integers less than 2020 relatively prime to 2020 is  $\phi_{2020} = 800$ . The set of record values for  $\sigma(m) + \phi(m)$  includes 2020, where record values are when the sum  $\sigma(m) + \phi(m)$  is greater than any previous values.

#### 4.17 Palindrome / Reverse Numbers

Adding 2020 to its palindrome 202 gives you the palindrome 2222. It is a plaindrome (non-decreasing) in base 11 and nialpdrome (nonincreasing) in base 14. Also, the divisors of 2020 reversed (i.e. 12 goes to 21) multiplied is 2020.

#### 4.18 Euler Transform

The Euler Transform of the powers of 5 includes the term 2020.

#### 4.19 Niven Number

2020 is a Super Niven Number, which means it is divisible by the sum of any subset of its nonzero digits.

#### 4.20 Undulating Number

2020 is a undulating numbers, which it has the form  $ABABA\dots$

#### 4.21 Gapful Number

2020 is a gapful number because it is divisible by its first and last digit.

#### 4.22 Zumkeller Number

2020 is a Zumkeller Number because its divisors can be partitioned into two sets with the same sum 2142.

#### 4.23 Iban Number

2020 is an iban number because when written as two thousand twenty it does not contain an "i".

#### 4.24 Self-Sliding Number

2020 is a self-sliding number because it can each digit  $d$  by  $d$  places either to the left or the right.

#### 4.25 Collatz Conjecture

It takes the number 2020 exactly 64 steps to reach 1.

#### 4.26 Abelian Squares

2020 is an abelian square, where an abelian square is a string of length  $2n$  where the last  $n$  symbols form a permutation of the first  $n$  symbols

#### 4.27 Continued Fractions

The set of the denominators of the continued fractions of  $\sqrt{102}$  and  $\sqrt{918}$  includes 2020.

#### 4.28 Other / Weird Representations

2020 can be written as  $n \cdot (5n + 1)$ , where  $n = 20$ . Also, 2020 can be represented as  $4n \cdot (4n^2 + 1)$ , where  $n = 5$ . Also, 2020 can be written as  $\frac{10n^2 + 4n + (1 - (-1)^n)}{8}$ , where  $n = 40$ . Also, 2020 is expressible as  $x^4 + y^2$ , and  $x^2 + 24y$  is an integer (in this case  $x = 4, y = 42$ ). Also, 2020 is expressible as  $\frac{n^3 + 9n^2 + 26n}{6}$ , where  $n = 20$ . Also, 2020 can be written as  $81n^2 - n$ , where  $n = 5$ . Also, 2020 can be written as  $25n^2 - 5$ , where  $n = 9$ . Also,  $\pi, e, \phi$  have the same digit in the 2020th decimal place. Also, 2020 can be written as  $\lfloor \frac{8^n}{7^n} \rfloor$ , where  $n = 57$ . The string 4,4 occurs in 2020 but not 2019 when using base 8. The string 8,4 occurs in 2020 but not 2019 when written in base 9.