Unorthodox Problems

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Computational Problems

Problem C1. Compute $0.1 + 0.02 + 0.003 + \ldots$.
Problem C2. 1st grader Nairit knows only the number 1. This means he can only repeatedly write down 1s, creating numbers such as $1, 11, 111, \cdots$; somehow, he manages to write down the smallest number of this form divisible by 2019. Let x be the number of 1s he wrote down. Find the remainder of x when divided by 1000.
Problem C3. Let a number be imajinary if it can be expressed as aj, where a is a real number, and $j = |\sqrt{2i}|$, where $i = \sqrt{-1}$. Let a number be khanplex if it can expressed as $-aj + bk$, where a, b are real number constants and $k = a + bi$ is complex. If a number x is imajinary and a number y is khanplex, $xy + x + y = zj$, and there exists only one value of $z = \frac{m}{n}$, find $m + n$.
Problem C4. There are 108 distinct heptominos, which are figures made out of seven unit squares that are connected to at least one other unit square by an edge. How many ways are there to arrange / connect these 108 heptominos such that they form a square?
Problem C5. There exists a natural number n whose square and cube contain exactly one of each of the numerals from 0 to 9. Find n.
Problem C6. In a group of males, there are exactly 2 father-son relationships, 2 uncle-nephew relationships, a grandfather-grandson relationship, and one elder and younger brother. What is the least number of people in this group? For example: If there were exactly 1 father and 1 son, it would be 2, because we would just need a father and a son.
Problem C7. If

$$x^2 + y^2 + z^2 + t^2 = 50,$$
$$y^2 + t^2 - x^2 - z^2 = 24,$$
$$xz = yt,$$
$$x + z + t = y,$$

Find x, y, z, t where $x, y, z, t \in \mathbb{R}$.

Proof Problems

Problem P1. Each point A in the plane is assigned a real number $f(A)$. Given $f(M) = f(A) + f(B) + f(C)$, where M is the centroid of $\triangle ABC$, prove that $f(A) = 0$ for all points A.

Problem P2. One glass contains 5 spoons of milk, and the other glass contains 5 spoons of tea. A spoon of milk was taken from the second glass and put in the first, then mixed thoroughly. Next, a spoon of tea (with milk) was poured back into the second glass. Is there more milk in the first glass or more tea in the second glass? Will the answer change after 10 such transfusions?

Problem P3. Given

$$a_n = an + b_{n=1}^{\infty},$$

$$a_n \cap F_n = \emptyset,$$

Find the minimum value of a and b, where F_n is the Fibonacci sequence.

Problem P4. Prove the following quantity diverges:

$$\frac{2}{3 - \frac{2}{3 - \frac{2}{\cdots}}}.$$

Problem P5. Compute

$$\sqrt{1 + 2\sqrt{1 + 3\sqrt{\ldots}}}.$$

Problem P6. For what value of x is the following maximum?

$$x^{x^{x^{\cdots}}}$$

Problem P7. Find n-digit positive integers A, B, C where

$$A + B = C.$$